Dualization of non-Abelian $B \wedge \varphi$ model

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In this work we show a dualization process of a non-Abelian model with an antisymmetric tensor gauge field in a three-dimensional space-time. We have constructed a non-Abelian gauge invariant Stückelberg-like master action, and a duality between a non-Abelian topologically massive $B \wedge \varphi$ model and a non-Abelian massive scalar action, which leads us to a Klein-Gordon-type action when we consider a particular case.

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I. INTRODUCTION

Interest has recently increased in dualization of non-Abelian theories. Indeed, Smailagic and Spallucci, in the framework of gauge models, have investigated a dualization of a non-Abelian $B \wedge F$ model in $D = 4$, in order to obtain a Stückelberg massive gauge invariant theory [1, 2, 3]. It is worth to mentioning that the dualization of topological models [4] as well as the interest for Stückelberg-like gauge invariant models is partly due to its relevance as alternatives to the Higgs mechanism for gauge fields mass generation [5, 6, 7].

The method used was first introduced by Buscher [8]. The procedure consists of gauging a symmetry of the original action by introducing non-propagating gauge fields and constraining the respective field strength to vanish by means of a Lagrange multiplier. After integrating over the Lagrange multiplier and fixing the gauge, we recover the original action. On the other hand, by integrating by parts the Lagrange multiplier term and then integrating out the gauge fields, we obtain the dual action.

In the framework of non-linear sigma models with non-Abelian isometries, the developments started with the work of de la Ossa and Quevedo [9], after a dual theory of an arbitrary sigma model with an Abelian isometry has been constructed [10]. More recently, Mohammedi shed new lights about this issue [11, 12].

Recently we have discussed an Abelian three-dimensional action with a topological term involving a two-form gauge field B and a scalar field, called for short $B \wedge \varphi$ term, in the framework of topological mass generation. Also we have showed that this action is related by Buscher's duality transformation to a massive gauge-invariant Stückelberg-type theory [13].

This is the only one model involving a Kalb-Ramond field defined in (2+1) dimensions. Besides the importance of dualities for string theories, the issue of broken symmetries restored at the quantum level could be enlightened by duality procedures. Indeed, a massive gauge invariant theory and a dual theory with the gauge symmetry broken may bring us interesting information at the quantum level.

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II.   THE ABELIAN MASTER ACTION

In this letter we will give the dualization of a non-Abelian version of the $B \wedge \varphi$ topological model. But it is instructive to analyse first the procedure for the Abelian theory, where the master action in three dimensions [13] is given by

$$S = \int_{M_3} \left\{ \frac{1}{2} H \wedge^* H + \frac{1}{2} (d\phi - U) \wedge^* (d\phi - U) + mB \wedge (d\phi - U) + m\Gamma \wedge dU \right\}. \tag{1}$$

In the expression above, U is the Stückelberg auxiliary vectorial field, Γ is the Lagrange multiplier field, and $H \equiv dB$ is a three form field-strength of the so-called Kalb-Ramond field B. The action (1) above has invariance under transformations

$$\delta B = d\Omega, \qquad \delta U = d\lambda, \qquad \delta\phi = \lambda, \qquad \delta\Gamma = d\alpha. \tag{2}$$

Variation of the action (1) above with respect to Γ gives the following field equation of motion:

$$dU = \alpha. \tag{3}$$

Therefore, from Poincaré's lemma, the auxiliary field is an exterior derivative of a zero form, namely,

$$U = d\phi'. \tag{4}$$

Putting the results (3) and (4) in the master action (1) and defining a new scalar field $\varphi = \phi - \phi'$, we have a $B \wedge \varphi$ model described by the action

$$S_{B\varphi} = \int_{M_3} \left\{ \frac{1}{2} H \wedge^* H + \frac{1}{2} d\varphi \wedge^* d\varphi + mB \wedge d\varphi \right\}. \tag{5}$$

Since ϕ' arises from the Stückelberg auxiliary field as given in the equation (4), a new definition of the scalar field ϕ causes no change in this physical problem. Therefore, the action (5) is gauge invariant under the following transformations

$$\delta B = d\Omega, \qquad \delta\varphi = 0. \tag{6}$$

Now, the variation in (1) with respect to the Kalb-Ramond field B implies that

$$d^* H - m (d\phi - U) = 0. \tag{7}$$

However, when we consider both the equation (4) and the definition of the scalar field φ in the equation (7) we obtain the equation

$$d (^* H - m\varphi) = 0, \tag{8}$$

whose general solution is given by

$$^* H - m\varphi = \Phi. \tag{9}$$

Inserting the former solution (9) in the master action (1) we find a massive model described by the action

$$S_\Phi = \int_{M_3} \left\{ \frac{1}{2} d\varphi \wedge^* d\varphi - \frac{m}{2} \varphi \wedge^* (m\varphi + \Phi) + \frac{1}{2} \Phi \wedge^* (m\varphi + \Phi) \right\}. \tag{10}$$

It is worth to mentioning that the scalar field Φ in the action (10) has no propagation. Besides, if we consider the particular case where $^*H - m\varphi$ is constant and zero, or in other words, if we consider the equations of motion, we obtain the Klein-Gordon massive model, namely

$$S_{KG} = \int_{M_3} \left\{ \frac{1}{2} d\varphi \wedge^* d\varphi - \frac{m^2}{2} \varphi \wedge^* \varphi \right\}, \tag{11}$$

where we have used

$$^*H = m\varphi, \tag{12}$$

which is the particular case mentioned above.

Therefore, from the master action (1) we have shown that the $B \wedge \varphi$ model (5) and the action for a massive scalar field (10) are dual to each other. Also we have shown that the action (10) leads to a free and massive Klein-Gordon action, when we consider the particular case where $^*H - m\varphi$ is constant and zero.

III.   THE NON-ABELIAN MASTER ACTION

Next, we shall see that there is a simple extension of the above procedure in the case of a non-Abelian internal symmetry. It is interesting to remark that the only possibility to construct a non-Abelian version of the master action (1) is via an introduction of an auxiliary vector field, as we have proved in Ref. [14], using the method of consistent deformations. Also, it is important to point out that the introduction of a one form gauge connection A is required to go further in the non-Abelian generalization of our model, although our original Abelian action (1) does not contain this field. Note that, as pointed out by Thierry-Mieg and Neeman [15] for the non-Abelian case, the field strength for B is

$$H = dB + [A, B] \equiv DB \tag{13}$$

Following Ref. [15], we can define a new $\mathcal{H}$ given by

$$\mathcal{H} = DB + [F, \Lambda] \tag{14}$$

where Λ is a one form auxiliary field and $F = dA + A \wedge A$.

The obstruction to the non-Abelian generalization lies only on the kinetic term for the antisymmetric field, but the topological term must be conveniently redefined. Therefore our non-Abelian master action can be written as [16]

$$S = \int_{M_3} \left\{ \frac{1}{2} \mathcal{H} \wedge^* \mathcal{H} + \frac{1}{2} (D\phi - U) \wedge^* (D\phi - U) + mB \wedge (D\phi - U) + m\Gamma \wedge DU \right\}, \tag{15}$$

The action (15) is invariant under the following transformations

$$\delta A = -D\theta, \quad \delta \phi = [\theta, \phi], \quad \delta B = D\Omega + [\theta, B], \quad \delta \Lambda = \Omega + [\theta, \Lambda], \tag{16}$$

where θ and Ω are zero and one form transformations parameters, respectively. The equation of motion with respect to the Lagrange multiplier field provides an analogous constraint to the obtained in the Abelian case (eq. (3)). This is possible only when a "flat connection" is imposed for the gauge vectorial field, that is $F(A) = 0$. Therewith, the variation of the model (15) regarding Γ supply us with

$$DU = 0. \tag{17}$$

Thus we have now

$$U = D\phi'. \tag{18}$$

Again the field ϕ' is a zero form as in the previous case (eq.(4)). Going back to the equation (15) with the results (17) and (18) and using the same former definition for a new scalar field $\varphi = \phi - \phi'$, that causes no change in the physical model, we obtain a non-Abelian $B \wedge \varphi$ model, namely,

$$S'_{B\varphi} = \int_{M_3} Tr \left\{ \frac{1}{2} \mathcal{H} \wedge^* \mathcal{H} + \frac{1}{2} D\varphi \wedge^* D\varphi - m\mathcal{H} \wedge \varphi \right\}. \tag{19}$$

Continuing with the same procedure of the Abelian case, we vary the action (15) with respect to the two form B and obtain

$$D\left(^*\mathcal{H} - m\varphi\right) = 0, \tag{20}$$

which leads us to

$$^*\mathcal{H} - m\varphi = \xi, \tag{21}$$

Again, inserting the solution (21) in the master action (15) we have a non-Abelian massive action, namely,

$$S'_\xi = \int_{M_3} Tr\left\{\frac{1}{2}D\varphi \wedge^* D\varphi - \frac{m}{2}\varphi \wedge^* (m\varphi + \xi) + \frac{1}{2}\xi \wedge^* (m\varphi + \xi)\right\}, \tag{22}$$

which is similar to the Abelian case.

Note that, analogous to the Abelian case, in the model (22) the scalar field ξ has not propagation and using the particular case where $^*\mathcal{H} - m\varphi$ is constant and zero we obtain a kind of non-Abelian Klein-Gordon massive model, namely

$$S'_{KG} = \int_{M_3} Tr\left\{\frac{1}{2}D\varphi \wedge^* D\varphi - \frac{m^2}{2}\varphi \wedge^* \varphi\right\}. \tag{23}$$

IV. CONCLUSIONS

In summary, we have shown in this letter a dualization process of a non-Abelian model in a three-dimensional space-time. We have constructed a non-Abelian gauge invariant master action, which generates a non-Abelian Stückelberg-like $B \wedge \varphi$ model and a non-Abelian massive action. For that, we have used the well established Buscher´s dualization method.

Kalb-Ramond fields arise naturally in string theory coupled to the area element of the two-dimensional worldsheet [17]. It is worthwhile to mentioning that duality between models involving Kalb-Ramond fields and scalar ones is rather rare in the literature.

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